Exam 4
Chapters 7,8 and 11

Answer the following questions on a separate piece of paper. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications.

Show your work!

1. Solve the triangle with angles $\alpha = 39.7°$ and $\gamma = 91.6°$ and side length $b = 16.4$. (10 points)

2. Solve the triangle with side lengths $a = 10$, $b = 5$ and $c = 8$. (10 points)

3. Explain in words why, given angle α and sides a and b (in the ambiguous case (SSA)), there are no triangles to solve if $a < h$ where h is the height of the triangle. (5 points)

4. A regular pentagon is inscribed in a circle of radius 10m. Find the length of a side of the pentagon. (Hint: Make some triangles and see what you can do.) (10 points)

5. Find the first five terms of the sequence given by $a_n = n^2 - 2$.(5 points)

6. Rewrite the formula for the sequence given by $a_n = \frac{(n+2)!}{(n-1)!}$ as products without factorials. (10 points)

7. Find the sum of the first 1000 numbers. (10 points)

8. Find the sum of the series (10 points)

$$\sum_{n=1}^{\infty} 2(\frac{1}{3})^{n-1}.$$

9. Convert the decimal 0.1666666... into a fraction.(No work, No credit)(Hint: Use a geometric series) (10 points)

10. Solve the following system of equations. (10 points)

$$5x - 6y = 23$$
$$x + 3y = 6$$

11. Solve the following system of equations. (10 points)

$$-2x + 5y = 14$$
$$7x + 6y = -2$$

Extra Credit:

1. A grocer wants to build a "mountain" of cans of soup. The first (bottom) level is to contain 9 rows of 12 cans each. Each level after the first is to contain one less row of 12 cans. Write a sequence whose terms are the number of cans at each level. Write the sum of the terms of this sequence in summation notation and find the number of cans in the mountain. (15 points)
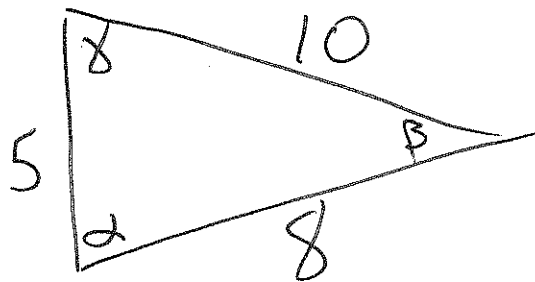
(1) $\alpha = 39.7°$, $\gamma = 91.6°$ and $b = 16.4$
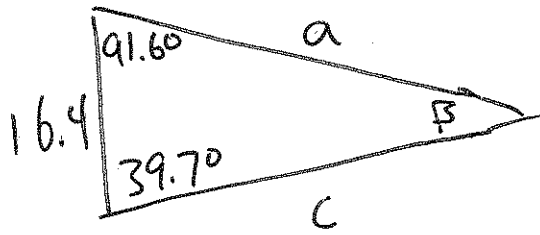
$\beta = 180° - 91.6° - 39.7° = 48.7°$

$$a = \frac{16.4 \sin 39.7°}{\sin 48.7°} = 13.94$$

$$c = \frac{16.4 \sin 91.6°}{\sin 48.7°} = 21.82$$

(2) $a = 10$, $b = 5$ and $c = 8$.

$$10^2 = 8^2 + 5^2 - 2 \cdot 8 \cdot 5 \cos\alpha$$
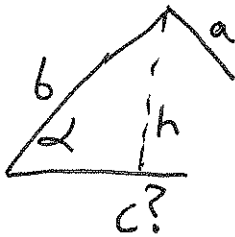
$$\cos\alpha = \frac{-11}{80} = -0.1375$$

$$\alpha = \cos^{-1}(0.1375) = 97.9°$$

$$\sin\beta = \frac{5\sin 97.9°}{10} = 0.4953$$
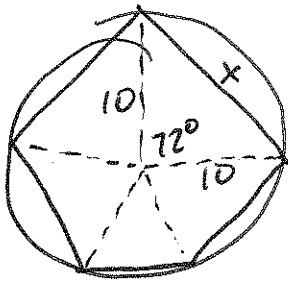
$$\beta = \sin^{-1}(0.4953) = 29.7°$$

$$\gamma = 180° - 97.9° - 29.7° = 52.4°$$

(3)

The side a cannot reach the side c.

(4)

$$\frac{360°}{5} = 72°$$

$$x^2 = 10^2 + 10^2 - 2(10)^2 \cos(72°)$$

$$x^2 = \cancel{\text{138.20}} \; 138.20$$

$$x = \cancel{\text{11.75}} \; 11.75\,m$$

(5)

$$a_n = n^2 - 2$$

$$a_1 = 1^2 - 2 = -1$$

$$a_2 = 2$$

$$a_3 = 7$$

$$a_4 = 14$$

$$a_5 = 23$$

(6)

$$a_n = \frac{(n+2)!}{(n-1)!} = \frac{(n+2)(n+1)(n)(n-1)(n-2)\cdots(1)}{(n-1)(n-2)\cdots(1)} = (n+2)(n+1)n.$$

(7)

$$1+2+3+\cdots+999+1000 = \sum_{n=1}^{1000} n = \frac{1000 \cdot (1001)}{2} = 500{,}50$$

(8)

$$\sum_{n=1}^{\infty} 2\left(\tfrac{1}{3}\right)^{n-1} = \frac{2}{1-\tfrac{1}{3}} = \frac{2}{2/3} = 3.$$

(9)

$$0.1666 = 0.1\bar{6} =$$

$$= 0.1 + 6 \times 10^{-2} + 6 \times 10^{-3} + 6 \times 10^{-4} + \cdots$$

$$= 0.1 + \frac{6 \times 10^{-2}}{1 - 10^{-1}} = \frac{6/100}{9/10} = \frac{1}{10} + \frac{2}{30} = \frac{5}{30} = \frac{1}{6}.$$

(10)

$$\begin{array}{l} 5x - 6y = 23 \\ 2(x + 3y = 6) \\ \hline 7x + 0y = 35 \end{array} \Rightarrow \begin{array}{l} 5(5) - 6y = 23 \\ -6y = -2 \\ y = \frac{1}{3}. \end{array}$$

$$x = \frac{35}{7} = 5$$

$$\{(5, \tfrac{1}{3})\}$$

(11)

$-2x+5y=14$ $\Rightarrow$ $x=\dfrac{5y-14}{2}$

$7x+6y=-2$

$7\left(\dfrac{5y-14}{2}\right)+6y=-2$

$\left(\dfrac{35}{2}+6\right)y-49=-2$ $\dfrac{47}{2}y=47$

~~$y=\dfrac{47}{...}$~~ $y=2$

$x=\dfrac{5(2)-14}{2}=-2$

$\{(-2,2)\}$

EC

(12)

$12\left\{\begin{array}{c}\circ\circ\cdots\circ\\ \vdots\\ \circ\circ\cdots\circ\end{array}\right.=108$ $+n\left\{\begin{array}{c}\circ\circ\cdots\circ\\ \vdots\\ \circ\cdots\circ\end{array}\right.=96$ $+\cdots+$ $12\left\{\begin{array}{c}\circ\\ \vdots\\ \circ\end{array}\right.=12$

$\displaystyle\sum_{n=1}^{9}12n$

$\displaystyle\sum_{n=1}^{9}12+(n-1)12$

~~$12\cdot108\cdot\dfrac{11\cdot108}{2}=648$~~

$\dfrac{9}{2}(12+108)=540$ cans